

Q1/8 Optics / Cable
Part (3)

Mansoura University	Faculty of Engineering	Electronics and Comm. Dept.	4 th Year, Optical Communication Systems	27/10/2015	Sheet #2
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Q1. Choose the correct answer

- 1-The values of the interatomic spacing for Si and Ge are ranging from
~~a) 3Å - 7Å~~ b) 3μm - 7μm c) 3nm - 7nm
- 2) Number of atoms in FCC is
a) one atom b) two atoms ~~c) 4 atoms~~
- 3) The wavelength emitted or absorbed for GaN having $E_g = 3.35\text{eV}$ is equal to
~~a) 370nm~~ b) 270nm c) 270μm
- 4) The bandgap energy for bulk structure is that of quantum well structures made from the same material
a) greater than ~~b) smaller than~~ c) equal to
- 5) Each allowed state has been described by quantum numbers
a) Two b) three ~~c) four~~
- 6) As the effective mass increases the density of states
~~a) increases~~ b) decreases c) doesn't change
- 7) As the effective mass increases the mobility of carriers
a) increases ~~b) decreases~~ c) doesn't change
- 8) As the frequency of collision increases, the carrier mobility
a) increases ~~b) decreases~~ c) doesn't change
- 9) The effective mass increases with the decrease in E for
a) bulk structures b) quantum well structures ~~c) Nano wire~~
- 10) If the effective mass is doubled, the density of state is in bulk material
~~a) multiplied by $2\sqrt{2}$~~ b) doubled c) halved
- 11) If the effective mass is doubled, the density of state is in Quantum Well
a) multiplied by $2\sqrt{2}$ ~~b) doubled~~ c) halved
- 12) As effective mass increases the curvature of E-K diagram
a) increases ~~b) decreases~~ c) doesn't change
- 13) The separation between two successive points along x-axis in K-space is equal to...
a) $2\pi/L_x$, L_x length of structure in x-direction ~~b) π/L_x , L_x length of structure in x-direction~~ c) L_x , L_x length of structure in x-direction

$$\lambda = \frac{hc}{E} = \frac{1241}{3.35} = 370 \text{ nm}$$

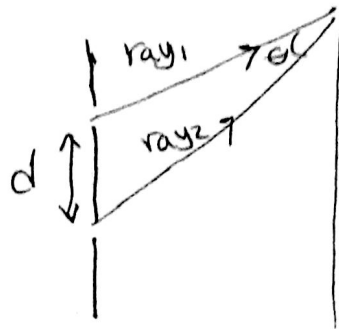
$$f \propto m$$

$$g \propto \frac{1}{E}$$

Sec (3)

E-k Diagram

→ Double Slit experiment



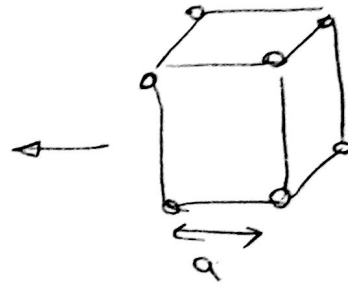
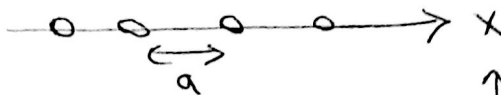
for ray₁ & ray₂ to make
Constructive interference

$$2d \sin \theta = n\lambda$$

Bragg's law

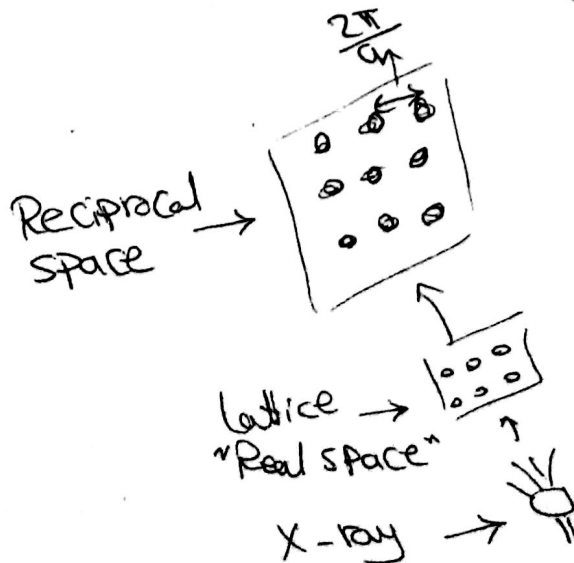
يوضح العلاقة بين distance الـ 2-slit و التداخل البناء والهدام

1D problem



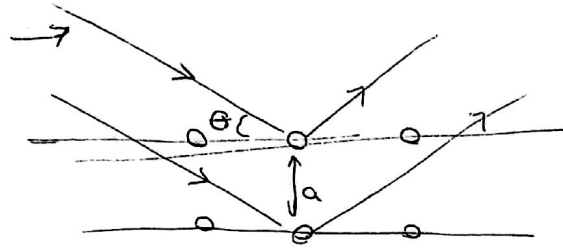
distance بين X و جارات مع distance
Real space ← (a) هنا

لكن واقعا هناك اقدر اسوف الـ lattice ← على X-rays
وبشوف دوره معكوس الـ lattice وليست دورتها
الحقيقية



↳ Critical ~~planes~~ lines where Bragg Condition occurs:

electron wave



$$2d \sin \theta = n\lambda$$

$$2a \sin \theta = n\lambda \quad \rightarrow \quad \frac{2\pi}{k}$$

$$a \sin \theta = n \frac{\pi}{k}$$

$$k = \frac{n\pi}{a \sin \theta} \quad \text{at } \theta = 90^\circ$$

↳ at the critical point of k
transmission of (electron) through the lattice wave

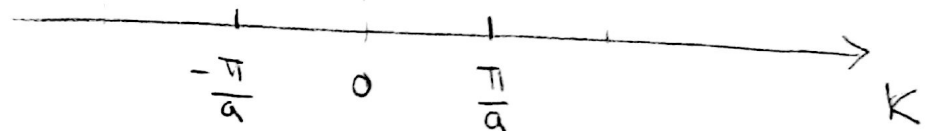
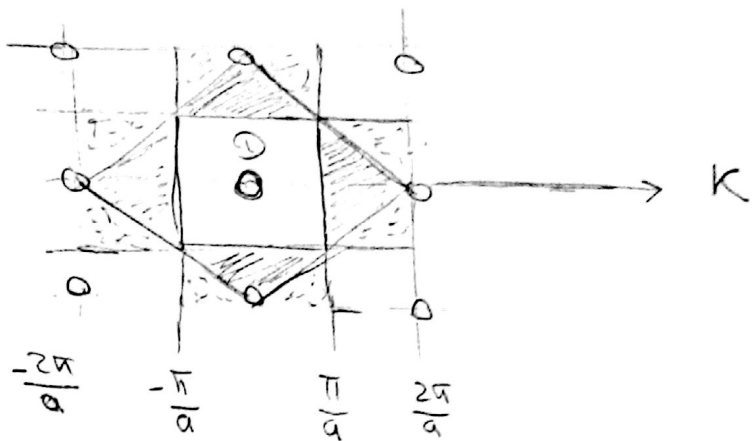
~~↳ no. of states = no. energy~~

↳ The Concept of no. transmission = energy Band gap

Reciprocal space (K-space)

① First Brillouin Zone

اقل مساحة ممكن اخذها
والتي بها يتكرر في الفراغ



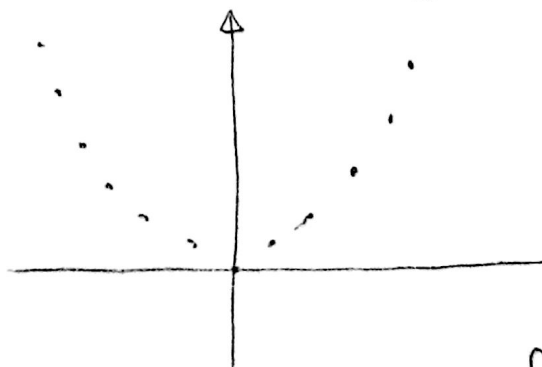
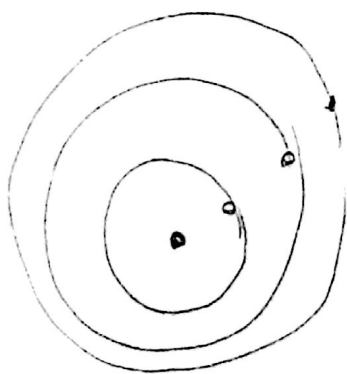
② لو قدرت اعرف شكل ال Energy مع ال K
في المنطقة دي هلاقيها بتكرر
بعد كده

Let's see the relation between E & K

↔ For free electron

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

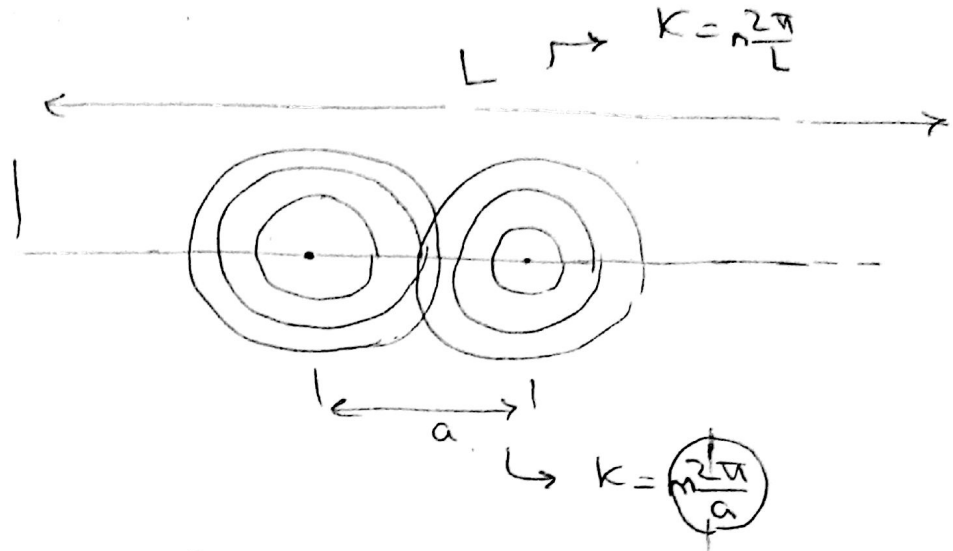
$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \frac{2\pi}{\lambda}$$



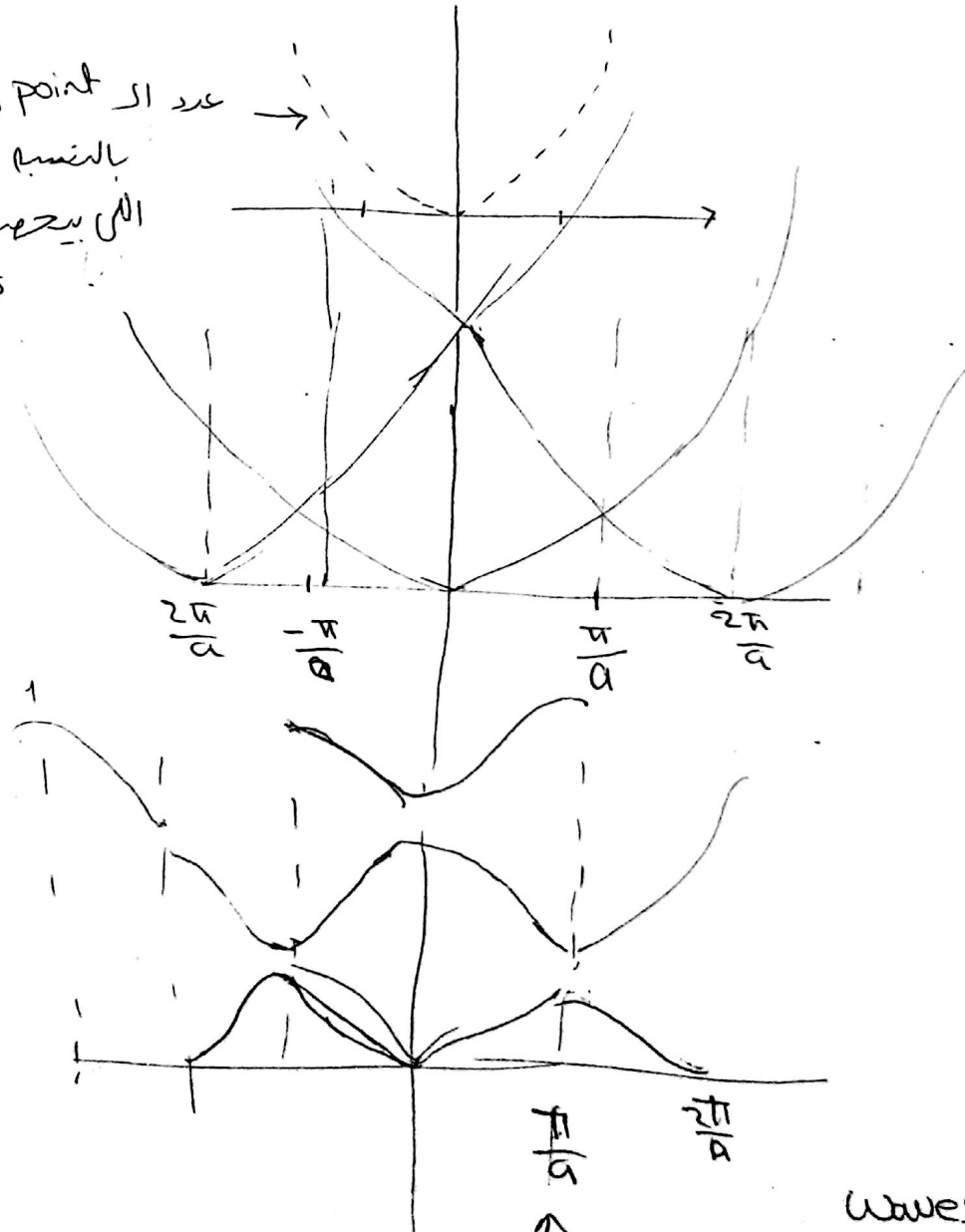
Discrete value of E → discrete value of k

3

↳ But in Solid



↳ In a solid at a point, the wave packets are overlapping and the energy levels are splitting into bands.



Waves for electron
 ↳ Will have Destructive Interference
 Bragg's Condition

Estimate the following relations

- Group velocity of electrons in solid
- Effective mass exhibited by electrons in solids
- Density of states $\rho_{c,v}(E)$ for 3D structures (bulk material)
- Density of states $\rho_{c,v}(E)$ for 2D structures (Quantum Well)
- Density of states $\rho_{c,v}(E)$ for 1D structures (Nano wire)

a) Group Velocity :

$$\text{Velocity} = \frac{p_{\text{req}}}{m} \rightarrow \frac{\hbar k}{m} \quad k = \frac{2\pi}{\lambda}$$

$$\text{Velocity} = v = \frac{d\omega}{dk} = 2\pi \frac{d f}{dk} \leftarrow p_{\text{req}}$$

From wave-particle

$$\text{Duality} \Rightarrow E = hf \Rightarrow dE = h df$$

$$df = \frac{dE}{h}$$

$$\therefore \boxed{v_g = \frac{2\pi}{h} \cdot \frac{dE}{dk}}$$

b) Effective mass

$\hookrightarrow$ In free space electron will move freely without any external force

$\hookrightarrow$ So, its mass = $m_0 = 9.1 \times 10^{-31}$ kg

$\hookrightarrow$ In Solid

Assume existence of electric force on (e^-)

energy $\rightarrow$ $dE = F \cdot dx$

$x \rightarrow dx \leftarrow$ displacement

$$dE = F \cdot v \cdot dt$$

$$dE = F \cdot \frac{2\pi}{h} \cdot \frac{dE}{dk} \cdot dt$$

$$\frac{dk}{dt} = F \cdot \frac{2\pi}{h} \Rightarrow F = \frac{h}{2\pi} \cdot \frac{dk}{dt} \rightarrow (1)$$

$$F = \overset{\substack{\uparrow \\ \text{effective} \\ \text{mass}}}{m^*} a \rightarrow (2); \text{ where } a = \frac{dU}{dt}$$

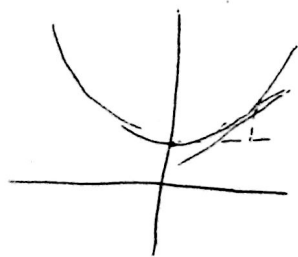
$$\therefore a = \frac{d}{dt} \left(\frac{2\pi}{h} \cdot \frac{dE}{dk} \right) \rightarrow (3)$$

From (1) & (2) & (3),

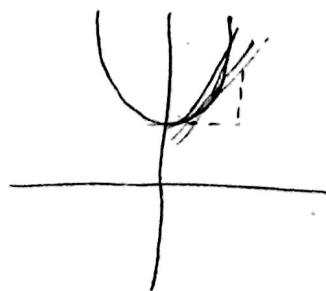
$$\frac{h}{2\pi} \cdot \frac{dk}{dt} = m^* \frac{d}{dt} \left(\frac{2\pi}{h} \cdot \frac{dE}{dk} \right)$$

$$\frac{h}{2\pi} = \frac{2\pi}{h} m^* \cdot \frac{d^2 E}{dk^2}$$

$$m^* = \frac{h^2}{(2\pi)^2} \cdot \frac{1}{d^2 E / dk^2} = \frac{h^2}{\underbrace{d^2 E / dk^2}_{\text{Curvature}}}$$



Low Curvature



High Curvature

Q4. Given

$$L_x = L_y = L_z = 1 \text{ mm} \rightarrow \text{Bulk material}$$

E

The cubic structure with 1mm side length and is made from materials with characteristics defined below.

$$E_2 \text{ (energy value in the conduction band)} = E_c + \frac{\hbar^2 (K-6)^2}{2m_c} \quad m_c = 0.98m_0 \quad \text{Si}$$

$$E_1 \text{ (energy value in the valence band)} = E_v - \frac{\hbar^2 K^2}{2m_v} \quad m_v = 0.49m_0 \text{ and } E_g = 1.1 \text{ eV}$$

a) Draw qualitatively the following curves:

i- E-K diagram and determine whether the considered material has direct or indirect bandgap and what is the indication of this classification?

ii- Density of states curves for conduction and valence bands and calculate the allowed states in the conduction band. From E_c to $E_c + 0.1 \text{ eV}$

b) Repeat the solution of (a) when the dimensions of structure doubled

c) Repeat the solution of (a) when the dimensions of the structure changed to be $1 \text{ mm} \times 1 \text{ mm} \times 10 \text{ \AA} \rightarrow \text{Quantum well}$

d) Repeat the solution of (a) when the values of m_c changed to $0.08 m_0$ and $m_v = 0.28 m_0$

$$a) \quad E_2 = E_c + \frac{\hbar^2 (K-6)^2}{2m_c} \rightarrow \text{at } K=6 \rightarrow E = E_c$$

$$E_1 = E_v - \frac{\hbar^2 K^2}{2m_v} \rightarrow \text{at } K=0 \rightarrow E = E_v$$

parabolic Approximation

E-K Diagram

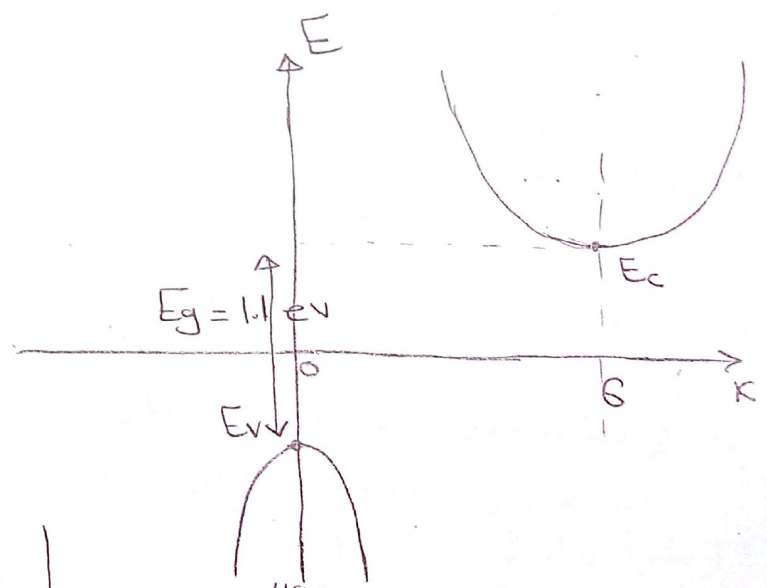
$$m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}}$$

Curvature of the Curve $\rightarrow \frac{d^2 E}{dk^2}$

if $m \rightarrow \text{large}$

Curvature $\rightarrow \text{small}$

if $m \rightarrow \text{small}$
Curvature $\rightarrow \text{large}$



$$0.98 m_0 > 0.49 m_0$$

$$m_c > m_v$$

Curvature of C.B. < Curvature of V.B.

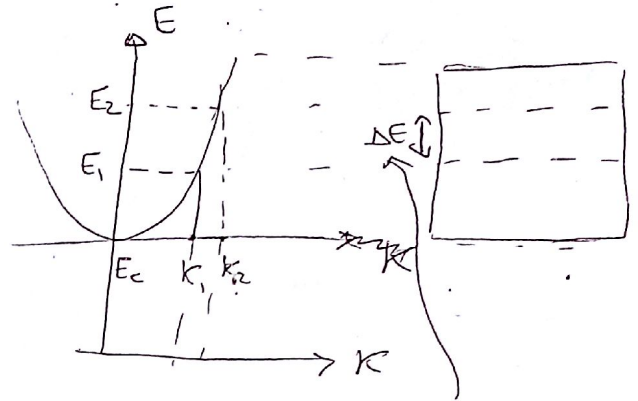
C] Density States $f(E)$ for 3D (Bulk material)

① Notes :

① Density $\rightarrow$ means anything per unit Volume

② E-K diagram

At every value of k there is Value for E



③ I Can use k -space

لأن لكل قيمة في k قيمة في E

لو قدرت اعرف عدد ال States
ال ممكن ال إلكترون بأخذها في المنطقة
دي و اعل احقا الي وجوده ممكن
او بصل Carrier Concentration
"تركيز الشحنات"
ال بحد كده هيفيدنا في معرفة التيار

↳ Derivation (3D)

↳ Assume free electron

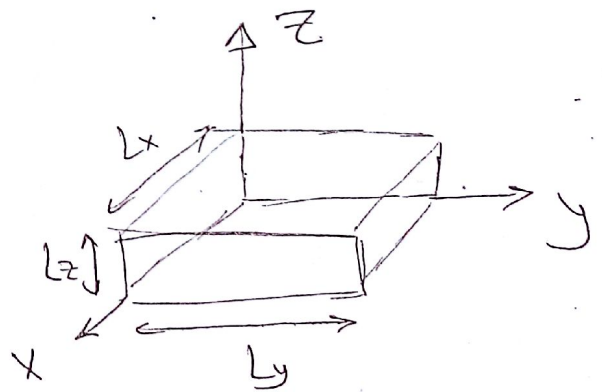
$$V = P.E = 0$$

طاقة الوضع

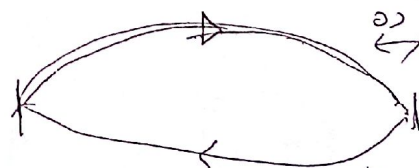
Particle in the Box

لكن في 3D

↳ Schrodinger eqn.



لو فيه إلكترون داخل هذه ال Material
ال Wave ال الناتجة عنه هتكون Standing wave
لو مش في الخارج



ال إلكترون يرجع تاني لأن بيكون موجة مرفوعة

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \psi(\vec{r}) + \left(\frac{2m}{\hbar^2}\right)(E - 0) \psi(\vec{r}) = 0$$

$$\psi(\vec{r}) = A e^{j\vec{k} \cdot \vec{r}} + B e^{-j\vec{k} \cdot \vec{r}}$$

dot product $\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$
 $\vec{r} = x \hat{x} + y \hat{y} + z \hat{z}$

تذکره:
 ملاکت بعدی 1D
 کت بفرضا
 $\psi(x) = A e^{ikx} + B e^{-ikx}$

بدل بفرستی ①
 (الو هو vector)
 $\therefore \psi(\vec{r}) = A e^{j(k_x x + k_y y + k_z z)} + B e^{-j(k_x x + k_y y + k_z z)}$

Apply B.Cs

$$\psi(0, 0, 0) = 0 \rightarrow ①$$

$$\psi(L_x, 0, 0) = 0 \rightarrow ②$$

$$\psi(0, L_y, 0) = 0 \rightarrow ③$$

$$\psi(0, 0, L_z) = 0 \rightarrow ④$$

From ①

$$0 = A e^0 + B e^0 \Rightarrow A = -B$$

From ②

$$\psi(\vec{r}) = A e^{j(\dots)} - A e^{-j(\dots)}$$

$$\cancel{A e} = A \sin(k_x x + k_y y + k_z z)$$

From ②

$$\psi(\vec{r}) = A \sin(k_x L_x) = 0$$

$$\therefore k_x L_x = m \pi$$

$$\hookrightarrow k_x = \frac{m \pi}{L_x}$$

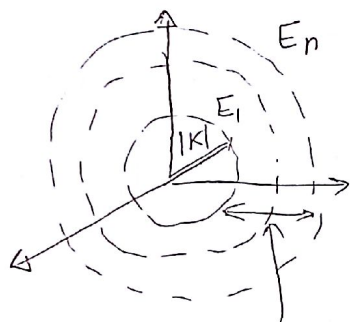
Similarly

$$k_y = \frac{p \pi}{L_y} \quad \& \quad k_z = \frac{q \pi}{L_z}$$

$\hookrightarrow$ To know no. of Allowed states
 $= \frac{\text{Volume of All states}}{\text{Volume of one state}}$

$\hookrightarrow$ Volume of one state
 $= \left(\frac{\pi}{L_x}\right) \left(\frac{\pi}{L_y}\right) \left(\frac{\pi}{L_z}\right)$

$\hookrightarrow$ Volume of all states
 $= \frac{4}{3} \pi k^3$



million of states in 3D

سیکونوا شکل کرہ

حجم الکرة $\frac{4}{3} \pi k^3$

قیمت ال (k) ص قَم discrete

اصغر حجم ممكن اخذ

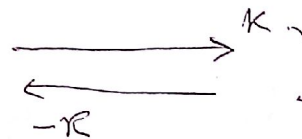
هو Cubic ابجاده

$\frac{\pi}{L_x} \& \frac{\pi}{L_y} \& \frac{\pi}{L_z}$

$\hookrightarrow$ لو اخذت حجم الكره كلها هأخذ ال $(-k)$ في الاعتبار
 لكن الاتجاه $(-k)$ هو اتجاه رجوع نفس الالكترون

Volume of all states

$= \frac{4}{3} \pi k^3 \left(\frac{1}{8}\right)$



الالكترون
 Standing wave

$\therefore$ No. of Allowed States $= \frac{\pi k^3 (L_x L_y L_z)}{6 (\pi^3)}$

No. of allowed States per unit Volume

$$= \frac{\pi k^3 (L_x L_y L_z)}{6(\pi^3) (L_x L_y L_z)}$$

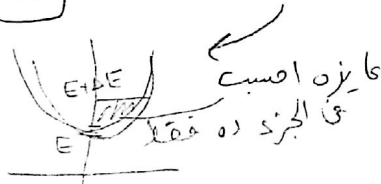
$$\boxed{N|_{\text{per unit Volume}} = \frac{k^3}{6\pi^2}}$$

لكن في ال State 2-electrons $\uparrow \downarrow$ spin $\uparrow \downarrow$
 ال State ال 2-electrons $\uparrow \downarrow$ spin $\uparrow \downarrow$

$$N|_{\text{per unit Volume}} = 2 \times \frac{k^3}{6\pi^2} = \frac{k^3}{3\pi^2}$$

total Allowed State per unit Volume

density of states per unit energy = $\frac{dN}{dE}$



$$= \frac{dN}{dk} \cdot \left(\frac{dk}{dE} \right) = \frac{dN}{dk} \cdot \frac{1}{dE/dk}$$

Conduction Band $\rightarrow$ الـ E الـ E+ΔE
 Valance Band $\rightarrow$ الـ E الـ E-ΔE

$$E = E_c + \frac{\hbar^2 k^2}{2m_c} \Rightarrow \frac{dE}{dk} = \frac{2\hbar^2 k}{2m_c}$$

$$E = E_v - \frac{\hbar^2 k^2}{2m_v} \Rightarrow \frac{dE}{dk} = \frac{2\hbar^2 k}{2m_v}$$

$$N = \frac{K^3}{3\pi^2} \Rightarrow \frac{dN}{dK} = \frac{3K^2}{3\pi^2} = \frac{K^2}{\pi^2}$$

For C.B/

$$f_c(E) = \frac{K^2}{\pi^2} \cdot \frac{1}{\hbar^2 K} \cdot m_c$$

$$= \frac{K m_c}{\pi^2 \hbar^2} \rightarrow (E - E_c)^{\frac{1}{2}} \left(\frac{2m_c}{\hbar^2} \right)^{\frac{1}{2}}$$

$$= (E - E_c)^{\frac{1}{2}} \left(\frac{2m_c}{\hbar^2} \right)^{\frac{1}{2}} \cdot \left(\frac{2m_c}{\hbar^2} \right) \cdot \frac{1}{2\pi^2}$$

$$\boxed{f_c(E) = \frac{1}{2\pi^2} (E - E_c)^{\frac{1}{2}} \left(\frac{2m_c}{\hbar^2} \right)^{\frac{3}{2}}}$$

$$f_v(E) = \frac{1}{2\pi^2} (E_v - E)^{\frac{1}{2}} \left(\frac{2m_v}{\hbar^2} \right)^{\frac{3}{2}}$$

→ The material is Indirect Bandgap.
Because V.B and C.B are at different $[K]$. [6]

[ii] For Bulk material [Density States Curves]

$$P_c(E) = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2}$$

$$P_v(E) = \frac{1}{2\pi^2} \left(\frac{2m_v}{\hbar^2} \right)^{3/2} (E_v - E)^{1/2}$$

$$P(E) \propto m^*$$

$$\therefore m_c > m_v$$

→ To Calculate all allowed states in the Conduction Band.

$$N = \int_{E_c}^{E_c + \Delta E} P_c(E) dE$$

V.B

assume $\Delta E = 0.1 \text{ eV}$

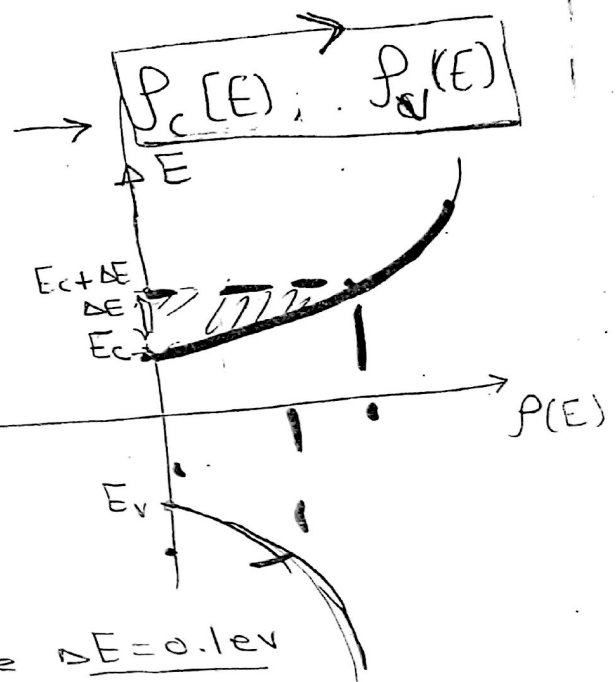
$$\therefore N = \int_{E_c}^{E_c + 0.1} \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2} dE$$

$$= \frac{2}{3} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \left(E - E_c \right)^{3/2} \Big|_{E_c}^{E_c + 0.1}$$

$$= \frac{2}{3\pi^2} \left(\frac{2 \cdot 0.98 m_0}{\hbar^2} \right)^{3/2} \left((E_c + 0.1) - E_c \right)^{3/2}$$

$$\hbar = 1.05 \times 10^{-34}$$

$$m_0 = 9.1 \times 10^{-31} \text{ kg}$$



$$N = \frac{1}{3\pi^2} \left(\frac{2 \cdot 0.98 \cdot 9.1 \cdot 10^{-31}}{(1.05 \cdot 10^{-34})^2} \right)^{3/2} \cdot (0.1 \cdot 1.6 \cdot 10^{19})^{3/2} \quad [7]$$

No. of state per unit Volume $3.16 \cdot 10^{25} \text{ m}^{-3} \approx 10^{19} \text{ cm}^{-3}$

↔ All no. of allowed state

$$= N \cdot \text{Volume}$$

$$= 10^{19} \cdot (1 \cdot 10^{-8})^3 \approx 10^{16} \text{ state}$$

[d] Repeat the solution of (a) when $m_c = 0.08 m_0$ & $m_v = 0.28 m_0$

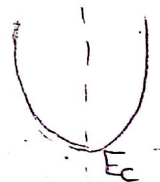
↔ $m_v > m_c$

Curvature | v.B < Curvature | c.B
 ↓
 "narrow" ~~band~~

[i] E-K diagram

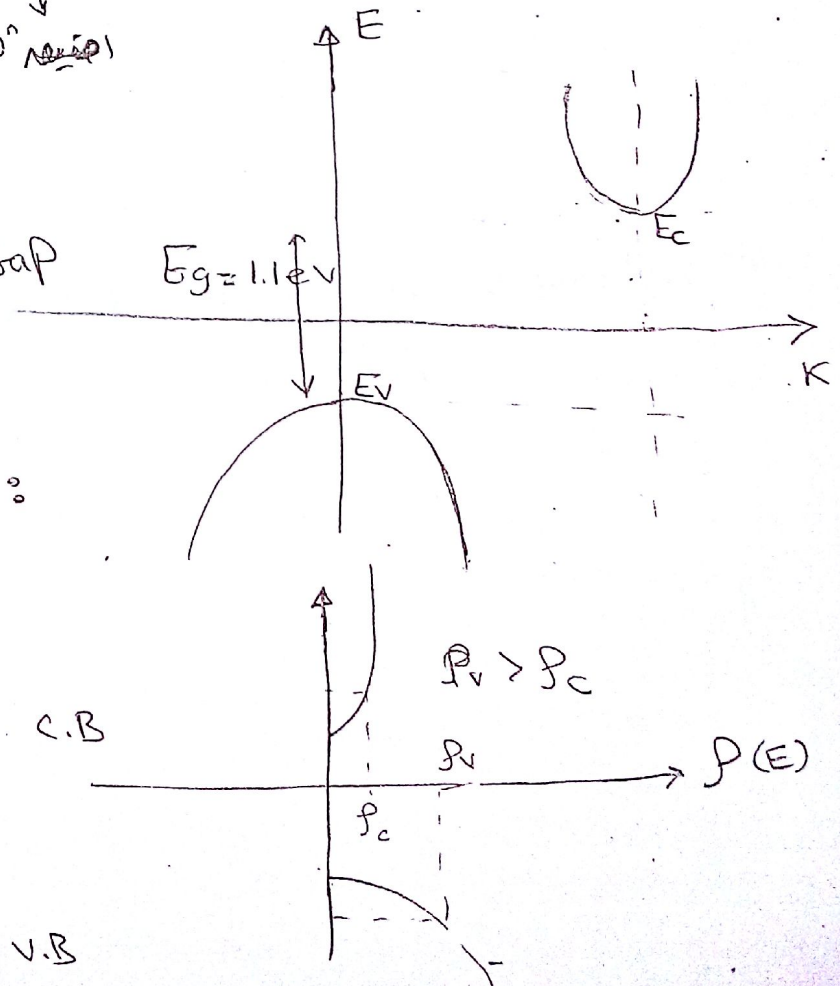
* Indirect Band Gap Material

$$E_g = 1.1 \text{ eV}$$



[ii] Density state Curve:

$f \propto m^*$
 $m_v \uparrow \uparrow \rightarrow f_v \uparrow \uparrow$
 $m_c \downarrow \downarrow \rightarrow f_c \downarrow \downarrow$



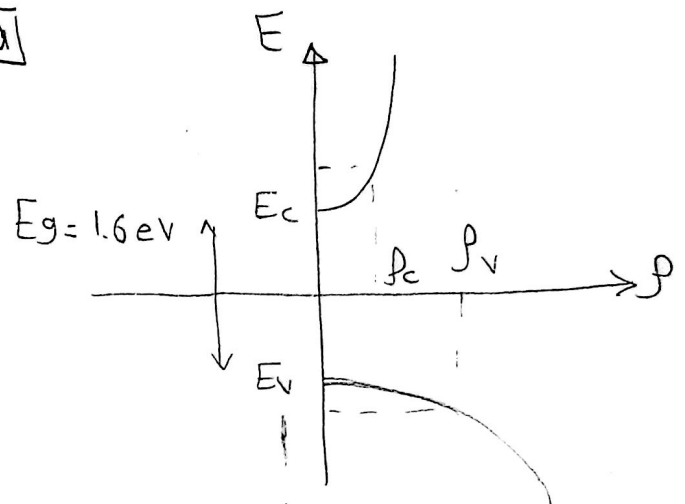
14 Draw E-K diagram qualitatively; for each material described by its density state.

a

$$\therefore p_v > p_c$$

$$m_v > m_c$$

Curvature of V.B < Curvature of C.B
 $\downarrow$
 "narrow"

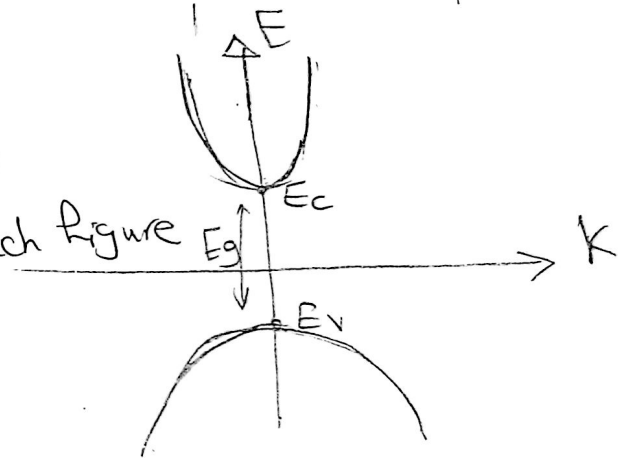


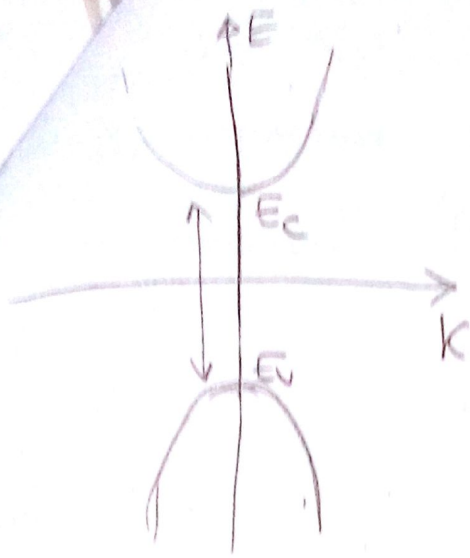
(c) Comment on the Carriers
 Mobility in each Figure

$$\mu = \frac{q\tau}{m^*}$$

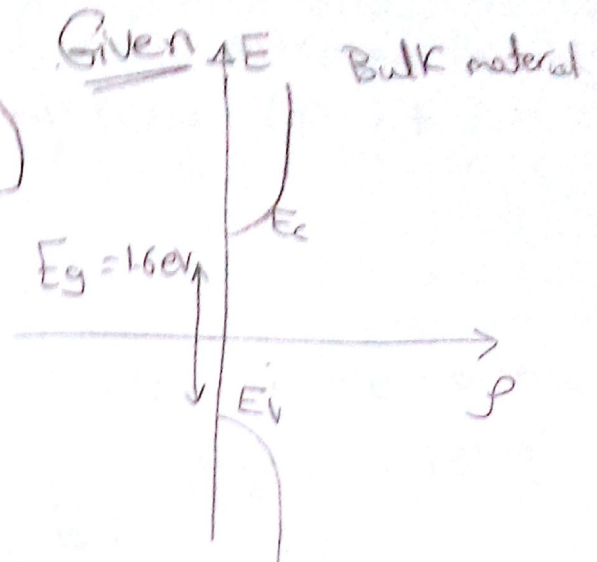
$$\mu \propto \frac{1}{m^*}$$

$M_{V.B} < M_{C.B}$ # Mobility of V.B is smaller than the C.B.





①
E-K Diagram



$$P_v \approx P_c$$



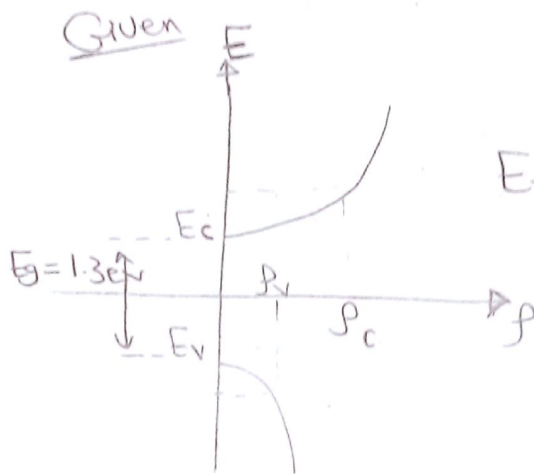
$$m_v \approx m_c$$

$$\sim \text{Curvature} \approx \text{Curvature C.B.}$$

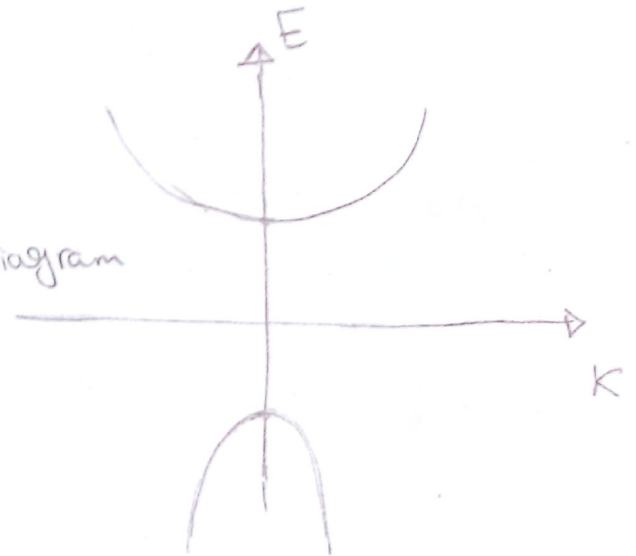
↪ (C)

Mobility of Carriers in C.B. is ~~the same as~~ Equal the mobility of V.B.

②



E-K Diagram



$$P_c > P_v$$

$$m_c > m_v$$

$$\text{Curvature C.B.} < \text{Curvature V.B.}$$

↓
(narrow)

Curve →

⇒ Mobility of Carriers in (electrons) C.B. is smaller than Carriers (holes) in V.B.

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Estimate the values of allowed states per unit Volume for structure represented by Curve (a) at energies

(a) $E_v - 0.1 \text{ eV}$

(b) $E_c + 0.1 \text{ eV}$

(c) E_c

assuming $m_c = 0.8 m_0$

Sol

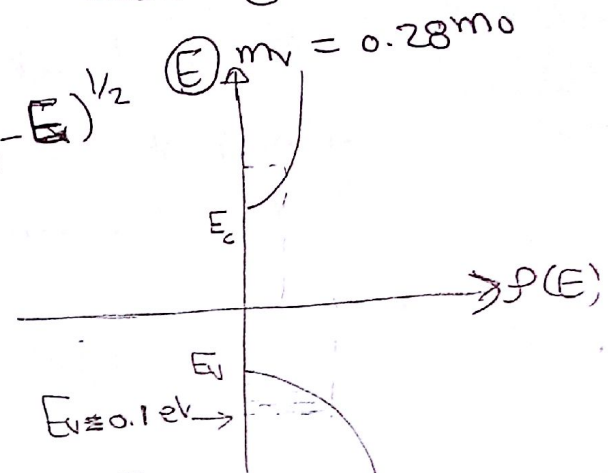
(a) $f_v(E) = \frac{1}{2\pi^2} \left(\frac{2m_v}{\hbar^2} \right)^{3/2} (E_v - E)^{1/2}$

$N_v = \int_{E_v - 0.1}^{E_v} f_v(E) dE$

$= \frac{1}{2\pi^2} \left(\frac{2m_v}{\hbar^2} \right)^{3/2} \left[\frac{2}{3} (E_v - E)^{3/2} \right]_{E_v - 0.1}^{E_v}$

$= \frac{1}{3\pi^2} \left(\frac{2 \times 0.28 \times 9.1 \times 10^{-31}}{(1.05 \times 10^{-34})^2} \right)^{3/2} \left(0.1 \times 1.6 \times 10^{-19} \right)^{3/2} \times 10^{-6}$

$= 2.26 \times 10^{19} \text{ cm}^{-3}$



(b) $f_c(E) = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2}$

$N_c = \int_{E_c}^{E_c + 0.1} f_c(E) dE$

$= \frac{1}{3\pi^2} \left(\frac{2 \times 0.8 \times 9.1 \times 10^{-31}}{(1.05 \times 10^{-34})^2} \right)^{3/2} \times (0.1 \times 1.6 \times 10^{-19})^{3/2} \times 10^{-6}$

$= 1.03 \times 10^{20} \text{ cm}^{-3}$

c) E_c :

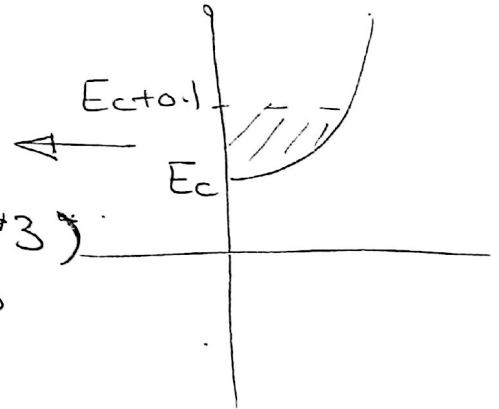
$$\rightarrow dE = 0 \rightarrow \boxed{N_c = 0}$$

no. of allowed state

h) Estimate the total no. of allowed states in the C.B assuming their dimensions are (1mm + 5mm + 3mm)

$$N_{\text{total}} = N_{\text{per unit volume}} \times \text{Volume}$$

$$N_c \approx 10^{20} \text{ cm}^{-3}$$



$$N_{\text{total}} \approx 10^{20} \times (1 \times 5 \times 3)$$

$$= 1.5 \times 10^{18} \quad \times 10^{-3}$$

$$\boxed{N_{\text{total}} \approx 10^{18}}$$

$$\uparrow \\ \text{mm}^3 \rightarrow \text{cm}^3$$